Shot-noise governed Coulomb blockade in a single Josephson junction

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We have investigated the influence of shot noise on the IV-curves of a single mesoscopic Josephson junction. We find that the blockade of the Cooper pair current is strongly suppressed in the presence of shot noise due to tunnelling in an adjacent SIN junction. Our experimental findings can be accounted for by an extension of the phase correlation theory. Shot-noise effect in a resistive environment R can be characterized by the effective noise temperature $T_N = eIR/2k_B$, which means that a Josephson junction can easily detect shot noise from a current well below 1 pA.

The role of quasiparticle tunnelling on the decoherence phenomena in Josephson junctions has become of great importance now when schemes for quantum computation are under development [1]. In the absence of dissipation from environmental modes, shot noise provides the ultimate dephasing mechanism at low temperatures. We have investigated the influence of external shot noise on a single Josephson junction in a strongly resistive environment, in which Coulomb blockade (CB) of Cooper pair current [2, 3] takes place owing to the delocalization of the phase variable [4].

The Coulomb blockade of Cooper pairs is very sensitive to fluctuations. Inherently, it is influenced by Johnson-Nyquist noise, which is predicted to result in a power-law-like increase of conductance both as a function of temperature and voltage [4]. The exponent of the power law, $2\rho - 2$, is governed by the parameter $\rho = R/R_Q$ where R describes the dissipative ohmic environment and $R_Q = h/4e^2$. Hence, in the case of large exponents $2\rho - 2 \gg 1$, there is a high resolution against tiny changes in temperature, or alternatively, a high sensitivity to any external noise sources.

In this Letter we report first investigations of "noise spectroscopy" using the Coulomb blockade of Cooper pairs as a sensitive detector. We have induced shot noise by a separately biased superconductor-insulator-normal metal (SIN) tunnel junction. The quasiparticle current is found to strongly reduce the CB of Cooper pairs: the influence can be resolved down to currents of a few pA in our experiments. Our findings can be well accounted for by including the effect of shot noise into the phase-phase correlation functions. To our knowledge, the present work is the first one to study quantitatively the effect of nonlinear dissipative elements on the Cooper pair tunnelling, as well as the first attempt to extend the phase-fluctuation theory [5] to account for an independent shot-noise source.

When the supercurrent channel is blocked off, the current is carried by incoherent tunnelling of Cooper pairs. Using $P(E)$-theory [6], the zero-bias conductance of the junction can be expressed via the real and the imaginary part of $J(t) = \langle[\varphi(t) - \varphi(0)]\varphi(0)\rangle = J_R(t) + iJ_I(t)$:

$$G_0 = \left.\frac{dI}{dV}\right|_{V\to 0} = -\frac{2e^2 E_J^2}{\hbar^3}\int_{-\infty}^{\infty} t\, dt e^{J_R(t)} \sin J_I(t)\,, \quad (1)$$

where the real part, $J_R(t) = J_T(t) + J_N(t)$, contains the contributions J_T from the equilibrium Johnson-Nyquist noise and J_N from the shot noise. Without the shot noise

$$J(t) = J_T + iJ_I = 2\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\frac{\mathrm{Re}Z(\omega)}{R_Q}\frac{e^{-i\omega t} - 1}{1 - e^{-\beta\hbar\omega}}\,. \quad (2)$$

For ohmic environment, $Z(\omega) = (1/R + i\omega C_T)^{-1}$ where C_T is the junction capacitance. The imaginary part $J_I(t) = -\pi\rho\left(1 - e^{-|t|/\tau}\right)\mathrm{sign}t$ does not depend on temperature and in the low-temperature limit $\hbar\beta = \hbar/k_B T \gg \tau$,

$$J_T(t) = 2\rho\left\{\frac{1}{2}\left[e^{-t/\tau}\mathrm{Ei}\left(\frac{t}{\tau}\right) + e^{t/\tau}\mathrm{Ei}\left(-\frac{t}{\tau}\right)\right] - \ln\left[\frac{\hbar\beta}{\pi\tau}\sinh\left(\frac{\pi t}{\hbar\beta}\right)\right] - \gamma\right\}, \quad (3)$$

where $\tau = RC_T$. The temperature dependence is important only at long times $t \gg \hbar\beta \gg \tau$, where temperature determines the phase diffusion, and $J_R(t) \approx -2\pi\rho t/\hbar\beta = -2\pi\rho k_B T t/\hbar$. Now we consider the contribution J_N of the shot noise. From the current–current spectral density $S_I = 2eI$ one can find the voltage-voltage spectral density:

$$S_V = |Z(\omega)|^2 S_I = \frac{2eIR^2}{1 + (\tau\omega)^2}\,. \quad (4)$$

Then using the Josephson relation $\hbar\partial\varphi/\partial t = 2eV$ one can obtain the phase-phase spectral density $S_\varphi = (4e^2/\hbar^2\omega^2)S_V$ and finally the contribution of the shot noise to J_R is

$$J_N(t) = \frac{\pi I}{e}\frac{R^2}{R_Q^2}\int_0^\infty \frac{d\omega}{\omega^2}\frac{\cos\omega t - 1}{1 + (\tau\omega)^2} = -\frac{\pi^2 I\tau}{2e}\frac{R^2}{R_Q^2}\left(e^{-|t|/\tau} + \frac{|t|}{\tau} - 1\right)\,. \quad (5)$$

As well as the thermal contribution, for small I the shot-noise contribution is significant only for large $t \gg \tau$

where the shot noise modifies the phase diffusion and $J_T(t) + J_N(t) = -2\pi\rho k_B(T + T_N)t/\hbar$, where $T_N = eIR/2k_B$ is the noise temperature.

Using the asymptotic expressions for $J_T(t)$ and $J_N(t)$ at $t \gg \tau$, one can derive an analytical expression for the conductance:

$$G_0 = \frac{2e^2E_J^2}{\hbar^3}\sin(\pi\rho)e^{A}e^{-2\rho\gamma_E}\left(\frac{2\pi\tau}{\hbar\beta}\right)^{2\rho-2}\tau^2 \times\frac{d\mathrm{B}(a/2+\rho,1-2\rho)}{da}, \qquad (6)$$

where $A = 2\pi k_B T_N\rho/\hbar$, $a = 2\rho T_N/T$, $\gamma_E = 0.577...$ is the Euler constant and $\mathrm{B}(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$ is the beta function. Using the functional relations between gamma functions, one can check that in the limit $T_N \to 0$ ($a \to 0$), Eq. (6) coincides with the conductance

$$G_0 = \frac{2\pi e^2E_J^2}{\hbar^3}e^{-2\rho\gamma}\left(\frac{2\pi\tau}{\hbar\beta}\right)^{2\rho-2}\tau^2\frac{\Gamma(\rho)}{\Gamma(2\rho)} \propto T^{2\rho-2} \quad (7)$$

calculated in Ref. [6] without the shot noise. In the opposite limit $T_N \gg T$ ($a \to \infty$) one can use the asymptotic relation $\Gamma(z + y) \sim \Gamma(z)z^y$ at $z \to \infty$, and Eq. (6) yields

$$G_0 = \frac{4\pi\rho\tau^2e^2E_J^2}{\hbar^3}\left(\frac{2\pi\rho\tau k_B}{\hbar}\right)^{2\rho-2}(T+T_N)^{2\rho-2} \quad (8)$$

for small ρ. In contrast to Eq. (7), Eq. (6) cannot be extended to large ρ, where the Coulomb blockade takes place and our experiment has been done. This is because times $t \sim \tau$ become relevant and the oscillating term $\sin(\pi\rho)$ is not cancelled at $a \neq 0$. But one may expect that the effect of shot noise still can be accounted for by the expression $G \sim (T + \gamma(\rho)T_N)^{2\rho-2}$, where $\gamma(\rho)$ may be considered as a $\rho-$dependent Fano-factor of order 1.

Our experiments were performed using a circuit layout which is depicted in Fig. 1. The physical structure consists of four basic elements: 1) an Al-AlO$_x$-Al Josephson junction (JJ) with a tunnel resistance of $R_T^{JJ} = 4-8$ kΩ, 2) a superconducting-normal Al-AlO$_x$-Cu tunnel junction (SIN) with $R_T^{SIN} = 6 - 27$ kΩ, 3) a thin film Cr resistor of $R_C = 23-67$ kΩ (20 μm long), located within a few μm from the Josephson junction, and 4) a similar Cr resistor R_B in front of the SIN junction. Altogether we investigated three samples, the parameters of which are given in Table I.

The circuits were fabricated using electron beam lithography and four-angle evaporation. The Cr resistor (5 nm thick, 100 nm wide) was evaporated first at an angle of $-18°$, followed by the Al-island at $-38°$. After oxidation, the sample holder was rotated by 45° around the z-axis and the SQUID-loop was deposited by a second Al-evaporation at $+38°$. Finally, the SIN-junction was made in copper deposition at $+6°$.

The JJ junction was, in fact, made of two 100×100 nm^2 junctions in a SQUID geometry in order to facilitate tuning of its Josephson energy. The Josephson energy E_J^{max} at no magnetic flux was calculated from the

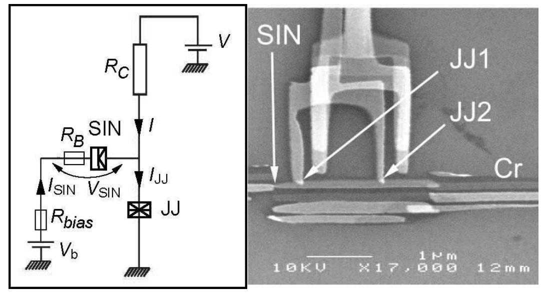

FIG. 1: A scanning electron microscope picture of sample 1 and a schematic view of the circuit. The chrome resistor is denoted by Cr, the superconductor-normal junction by SIN, and the Josephson junction in a SQUID-loop configuration by JJ1 and JJ2. For constructional details, see text.

	R_T^{JJ}(kΩ)	R_T^{SIN}(kΩ)	R_C(kΩ)	R_B(kΩ)	E_C	E_J^{min} / E_J^{max}
1	8.1	27.3	22.6	0.1	65	22 / 78
2	7.8	5.8	54.2	0.1	50	83 / 83
3	4.3	10	67	53	35	14 /150

TABLE I: Device parameters for our three samples numbered consecutively by the first column. The next two columns give the tunnelling resistance of the Josephson junction R_T^{JJ} and the SIN junction R_T^{SIN}. R_B and R_C denote the impedances in the immediate vicinity of the SIN and Josephson junctions, respectively. The last two columns indicate the Coulomb energy, E_C, and the minimum E_J^{min} and maximum E_J^{max} values of the Josephson energy. The energies are given in μeV.

tunnelling resistance using the Ambegaokar-Baratoff relation. The minimum Josephson coupling energy, E_J^{min}, was obtained from the minimum of critical current $I_C(\Phi)$ as a function of external flux Φ, and assuming a linear dependence between E_J and I_C. The Coulomb energy $E_C = e^2/2C$ was estimated from the IV-curves in the normal state: the sum of junction capacitances, $C = C_{SIN} + C_{JJ}$, was obtained from the voltage offset at large bias voltages using the formula $V_{\mathrm{offset}} = \frac{e}{2C}$. The ratio E_J/E_C could be tuned over the range 0.33 and 4.3 (see Table I). External noise was filtered out by 1.5 MHz low-pass filters at the top of the cryostat and by 1 m of Thermocoax cable at the mixing chamber.

Fig. 2a displays the temperature dependence of zero-bias conductance $G_0(T) \propto T^{(2\rho-2)}$, measured on sample 3. Using $R = R_C = 67$ kΩ, we get for the exponent $2\rho - 2 = 19$ which yields the power law shown by the solid curve in Fig. 2. This verifies that the steep, measured $G_0(T)$ of our sample agrees quite well with the theoretical temperature scaling law of Eq. (7) at the lowest temperatures. Slightly better agreement was found for sample 1 where the condition for the validity of the theory, $\rho k_B T < E_C$, is easier to fulfill.

As a second test of theory, we show the dependence of G_0 on the Josephson energy E_J in Fig. 2b. At low values of E_J we recover the expected E_J^2 dependence. Hence, we

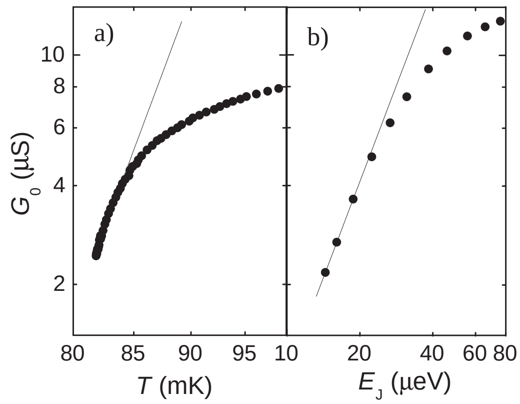

FIG. 2: a) Zero bias conductance $G_0 = \frac{dI_{JJ}}{dV}|_{I_{JJ}=0}$ for the JJ + R_C section of sample 3 as a function of temperature T at $E_J = 14$ μeV. The solid curve illustrates Eq. (7) using the ohmic environment of $R = R_C$. b) G_0 as a function of Josephson energy E_J at $T = 82$ mK. The solid curve illustrates the quadratic dependence obtained from Eq. (7).

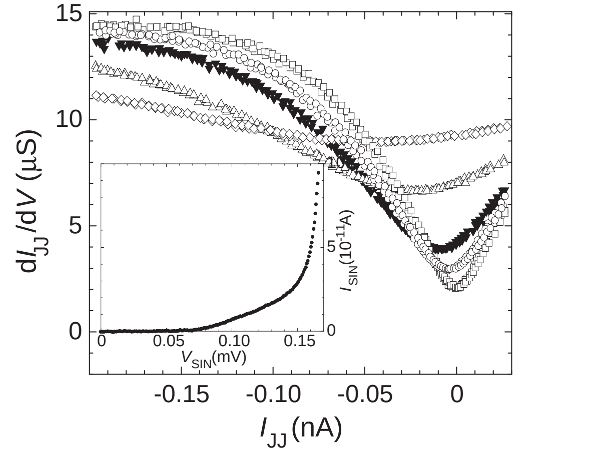

FIG. 3: Differential conductance $\frac{dI_{JJ}}{dV}$ vs. current I_{JJ} for the JJ + R_C section of sample 3 with a small current bias (via 100 MΩ resistor) in the SIN junction: $I_{SIN} = 0$ ($\square$), 0.01 ($\circ$), 0.02 ($\blacktriangledown$), 0.05 ($\triangle$), 0.1 nA ($\Diamond$). The inset shows the I_{SIN} vs. V_{SIN} for the SIN-junction biased via JJ (R_C was employed as the voltage lead). $T = 82$ mK.

conclude that, in the small E_J limit, the conduction in the JJ is predominantly caused by inelastic Cooper pair tunnelling, and no extra leakage is present at the lowest temperatures. Under these circumstances, any change in G_0 can be assigned to an additional external source of noise in the circuit.

The Coulomb blockade at small voltages is seen most clearly using measurements of differential conductance $\frac{dI}{dV}$. Fig. 3 displays the measured $\frac{dI_{JJ}}{dV}$ for sample 3 at zero quasiparticle current as well as at a few values of I_{SIN} ranging from 0.01 nA to 0.1 nA; the data was taken at the minimum value of $E_J = 14$ μeV. The bias voltage V to the JJ was applied via the chrome resistor while the SIN-junction was current biased through $R_{bias} = 100$ MΩ. The minimum amplitude of the Coulomb blockade dip $G_{min} = \left[\frac{dI_{JJ}}{dV}\right]_{min}$ is seen to increase monotonically with I_{SIN}. In addition, there is a small shift by $\Delta I_{JJ} \sim -0.20 \cdot I_{SIN}$ in the location of the minimum conductance with increasing I_{SIN}. Sample 1 yielded $\Delta I_{JJ} \sim -0.22 \cdot I_{SIN}$, but in sample 2, the large value of E_J prevented any quantitative analysis.

The IV-curve (I_{SIN} vs. V_{SIN}, see Fig. 1) of the SIN junction is illustrated in the inset of Fig. 3. The biasing was applied through JJ while the voltage was recorded via R_C. In the subgap region, there is only a small current, presumably due to Andreev reflection processes [7]. At the gap edge, the current increases rapidly and the differential resistance $R_d = \frac{dV_{SIN}}{dI_{SIN}}$ drops down to 5 - 10 kΩ at currents 0.05 nA - 1 nA. This means that the resistive environment R seen by the Josephson junction varies with the biasing of the SIN junction. When $I_{SIN} < 20 - 30$ pA, $R_{SIN} \gg R_C$ and the environment is purely governed by R_C. On the other hand, in the regime $0.05 - 1$ nA, we may approximate $R^{-1} \sim R_C^{-1} + (R_B + R_d)^{-1}$. Here, we neglect all the second order terms which might give a noticeable contribution to the dissipative part of the impedance [8].

In addition to shot noise, the SIN junction might give a contribution via the Johnson–Nyquist noise, which is determined by the differential resistance R_d at the biasing point. Since R_d is not monotonous and has a minimum as a function of I_{SIN}, this would result in a nonmonotonous current dependence for the Coulomb blockade. None of our samples, however, showed any nonmonotonous behavior. The absence of any re-entrant type of behavior supports our observation that we are dealing with the effect of shot noise.

As there are uncertainties in the theory, we deduce T_N by equating $G_0(I_{SIN})$ with $G_0(T + T_N)$. The results are displayed in Fig. 4 for samples 1 and 3. Both samples show a nearly linear increase in T_N with growing I_{SIN}. A comparison with the formula $T_N = eIR/2k_B$, yields for the Fano-factor $\gamma = 3$ and 0.5 for the samples 1 and 3, respectively. The inset of Fig. 4 displays the I_{SIN}-dependence of G_0 for sample 3. A fit using Eq. (8) with a "high-current" environment of $R^{-1} = R_C^{-1} + R_B^{-1}$ yields $\gamma \sim 1$.

The measured Fano-factors, at least qualitatively, agree with our theoretical scenario, which up to now has not yet provided numerical values for the Fano factor at large ρ. But there are additional, neglected physical processes, which also can account for the deviation of the measured Fano-factors from 1 [9]. Especially, there is an uncertainty in the subgap regime of the SIN junction: the conduction should be caused only by Andreev processes. This would indicate that the Fano factor should be 2 for I_{SIN} (tunnelling of 2e charges). At larger cur-

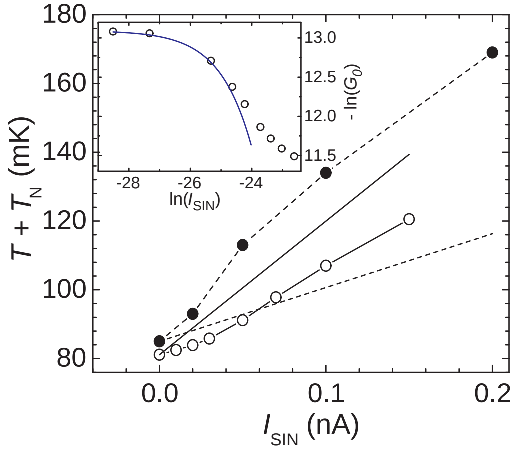

FIG. 4: Temperature $T+T_N$ vs. I_{SIN} obtained by comparing $G_0(I_{SIN})$ with $G_0(T)$: sample 1 (• − −−) and sample 3 (○ ——). The dashed and solid lines illustrate the formula $eI_{SIN}R/2k_B$ for samples 1 and 3, respectively. Inset: G_0 of sample 3 as a function of I_{SIN}. The fit is obtained using Eq. (8) with $R^{-1} = R_C^{-1} + R_B^{-1}$ in order to have a better environment for the high current regime.

rents, the dynamic impedance of the SIN junction is expected to lower the exponent ρ. Last but not least, there is the question of correlations between quasiparticle and Cooper pair tunnelling.

Our model assumes additivity of thermal and shot noise and, therefore, cannot account for the shift of the minimum of conductance to finite current values by ΔI_{JJ} in Fig. 3. But one should not rule out the possibility of two correlated noise sources, one on SIN and one on JJ. Phenomenologically, we may write

$$T_{eff} = T + \frac{eR}{2k_B}\left[\gamma I_{SIN} + \kappa 2 I_{JJ} + \lambda\sqrt{2\gamma\kappa I_{SIN} I_{JJ}}\right] \tag{9}$$

where κ denotes the Fano factor of the Cooper pair tunnelling noise in the Josephson junction and λ describes the correlations between the two noise sources. By minimizing this with respect to I_{JJ} at a fixed I_{SIN}, we obtain

$$I_{JJ} = \frac{\gamma\lambda^2}{8\kappa} I_{SIN} \tag{10}$$

which defines the condition for the minimum T_{eff} (with $\lambda < 0$).

Using the experimental value of γ and the shift ΔI_{JJ} for sample 3, we get an upper limit $\kappa < 0.31$ (at maximum correlation $\lambda = -1$). We expect this limit to hold for sample 1 as well: then, by taking $\kappa = 0.31$, we obtain $\lambda = -0.43$. Thus, it appears that the correlations between I_{JJ} and I_{SIN} depend on the base resistor and that an increase in R_B tends to suppress uncorrelated tunnelling current in the SIN junction.

According to our experimental analysis, a single Josephson junction provides a good candidate for a noise detector in general. Its main virtue is the high sensitivity which comes from the large detector band width: $\sim 1/RC$. Experimentally, changes by 1 mK in T_N can be resolved clearly in Fig. 4. This corresponds to a quasiparticle current of $I_{qp} = 3$ pA, which equals the sensitivity in high-resolution noise experiments of Ref. [10]. An enhancement by a factor of ten in the noise sensitivity is obtainable by improving the stability of $\frac{\Delta R}{R}$ measurement to the level of 1% and working at $T = 20$ mK. This would allow, for example, detailed studies of the back-action noise of quantum amplifiers such as superconducting SETs. A larger dynamic range can be obtained by operating the device at lower T.

In summary, our measurements of G vs. I curves for solitary, resistively confined small Josephson junctions show that the Cooper pair blockade can be suppressed strongly by shot noise from a near-by SIN tunnel junction. Using the framework of the phase correlation theory, we have presented a theoretical analysis of the shot-noise effect, which can be characterized by an effective noise temperature of the system T_N. This approach yields a good agreement with our experimental results. We expect that other sources of noise ($1/f$–noise, as an example), produce similar effects. Consequently, CB of Cooper pairs can be employed as a sensitive noise detector with a resolution of 0.1 mK in T_N.

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